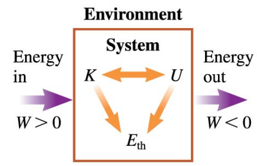
Physics 4A
Chapters 9: Work and Kinetic Energy

MODEL 9.1

Basic energy model

Energy is a property of the system.



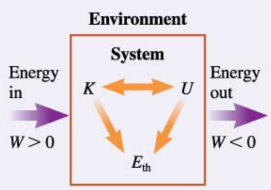
- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces from the environment.
  - The forces do *work* on the system.
  - $W > 0$ for energy added.
  - $W < 0$ for energy removed.
- The energy of an *isolated system*—one that doesn't interact with its environment—does not change. We say it is *conserved*.
- The energy principle is $\Delta E_{sys} = W_{ext}$.
- Limitations: Model fails if there is energy transfer via thermal processes (heat).

Exercise 1

GENERAL PRINCIPLES

Basic Energy Model

- Energy is a property of the system.
- Energy is *transformed* within the system without loss.
- Energy is *transferred* to and from the system by forces that do work W.
- $W > 0$ for energy added.
- $W < 0$ for energy removed.



The Energy Principle

Doing work on a system changes the **system energy:**

$$\Delta E_{sys} = W_{ext}$$

For systems containing only particles, no interactions, $E_{sys} = K + E_{th}$. All forces are external forces, so

$$\Delta K + \Delta E_{th} = W_{tot}$$

where W_{tot} is the total work done on all particles.

IMPORTANT CONCEPTS

Kinetic energy is an energy of motion: $K = \frac{1}{2}mv^2$

Potential energy is stored energy.

Thermal energy is the **microscopic** energy of moving atoms and stretched bonds.

Dissipative forces, such as friction and drag, transform **macroscopic** energy into thermal energy. For friction:

$$\Delta E_{th} = f_k \Delta s$$

The **work** done by a force on a particle as it moves from s_i to s_f is

$$W = \int_{s_i}^{s_f} F_s\, ds = \text{area under the force curve}$$

The work done by a constant force is

$$W = \vec{F} \cdot \Delta \vec{r}$$

The work done by a spring is

$$W = -\left(\tfrac{1}{2}k(\Delta s_f)^2 - \tfrac{1}{2}k(\Delta s_i)^2\right)$$

where Δs is the displacement of the end of the spring.

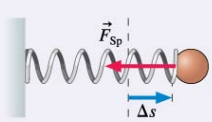
APPLICATIONS

Hooke's law

The **restoring force** of an ideal spring is

$$(F_{Sp})_s = -k\,\Delta s$$

where k is the **spring constant** and Δs is the displacement of the end of the spring from equilibrium.



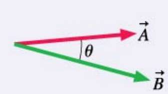
Power is the rate at which energy is transferred or transformed:

$$P = dE_{sys}/dt$$

For a particle with velocity $\vec{v}$, the power delivered to the particle by force $\vec{F}$ is $P = \vec{F} \cdot \vec{v} = Fv\cos\theta$.

Dot product

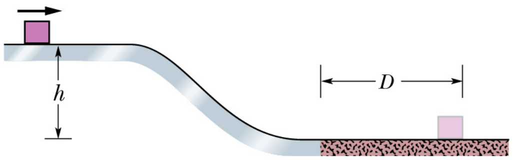
$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_xB_x + A_yB_y$$



Question and Example Problems from Chapter 9

Conceptual Question 9.A

In the figure below, a block slides along a track that descends through distance h. The track is frictionless except for the lower section. There the block slides to a stop in a certain distance D because of friction. **(a)** If we decrease h, will the block now slide to a stop in a distance that is greater than, less than, or equal to D? **(b)** If instead, we increase the mass of the block, will the stopping distance now be greater than, less than, or equal to D?



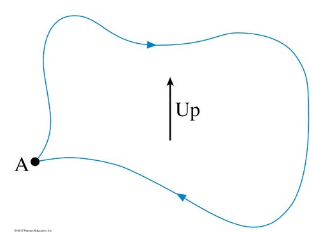
Conceptual Question 9.3

An elevator held by a single cable is descending but slowing down. Is the work done by tension positive, negative, or zero? What about the work done by gravity? *Explain.*

9.3. The work done by the tension in the cable is negative because the load's displacement is downward and the force is upward. The angle between them is $180°$ so the cosine of the angle is negative. The work done by the gravitational force is positive because the force is down and the displacement is down. The angle between them is zero so the cosine is positive.

Conceptual Question 9.7

A particle moves in a vertical plane along the closed path seen in the figure, starting at A and eventually returning to its starting point. Is the work done by gravity positive, negative, or zero.



9.7. No work was done by gravity. $W_g = -m_g \Delta y$. Here, $\Delta y = 0$. Any work done during a downward part of the motion was undone during the upward parts.

Conceptual Question 9.8

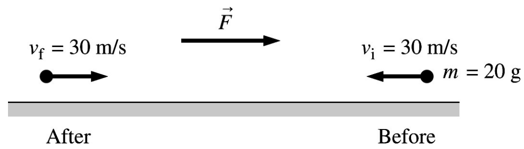
A need to raise a heavy block by pulling it with a massless rope. You can either **(a)** pull the block straight up height h, or **(b)** pull it up a long, frictionless plane inclined at a 15 angle until its height has increased by h. Assume you will move the block at constant speed either way. Will you do more work in case a or case b? Or is the work the same in both cases? Explain.

9.8. The work is the same in both cases, since the work done against gravity is $-m_g \Delta y$, and Δy, the change in height, is the same in both cases.

Problem 9.7

A 20 g particle is moving to the left at 30 m/s. A force on the particle causes it to move to the right at 30 m/s. How much work is done by the force?

9.7. Model: Use the work–kinetic energy theorem to find the net work done on the particle.
Visualize:



Solve: From the work–kinetic energy theorem,

$$W = \Delta K = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m(v_1^2 - v_0^2) = \frac{1}{2}(0.020 \text{ kg})\left[(30 \text{ m/s})^2 - (-30 \text{ m/s})\right]^2 = 0 \text{ J}$$

Assess: Negative work is done in slowing down the particle to rest, and an equal amount of positive work is done in bringing the particle to the original speed but in the opposite direction.

Problem 9.10

The cable of a crane is lifting a 750 kg girder. The girder increases its speed from 0.25 m/s to 0.75 m/s in a distance of 3.5 m. **(a)** How much work is done by gravity? **(b)** How much work is done by tension?

9.10. Model: Model the girder as a particle with no air resistance.
Visualize: We are given $v_{0y} = -4.0$ m/s and $y_0 = 35$ m, and $y_1 = 0$ m.
Solve: (a) The gravitational force exerted by the earth is $F_G = mg$. The work done is then

$$W = \int_{y_0}^{y_1} F_y dy = F_y \Delta y = -mg\Delta y = -(750 \text{ kg})(9.8 \text{ m/s}^2)(3.5 \text{ m}) = -25725 \text{ J}$$

We report this as –26 kJ.
(b) The net (total) work done on the girder is its change of kinetic energy.

$$\Delta K = \tfrac{1}{2}m(v_f^2 - v_i^2) = \tfrac{1}{2}(750 \text{ kg})((0.75 \text{ m/s})^2 - (0.25 \text{ m/s})^2) = 187.5 \text{ J}$$

The work done by tension is the total work minus the work done by gravity.

$$W_{\text{tension}} = W_{\text{total}} - W_{\text{grav}} = 187.5 \text{ J} - (-25725 \text{ J}) = 25900 \text{ J}$$

We report this as 26 kJ.
Assess: The magnitudes of the answers are the same because we rounded both to two significant figures.

Problem 9.11

Evaluate the dot product $\vec{A} \bullet \vec{B}$ if

(a) $\vec{A} = 4\hat{i} - 2\hat{j}$ *and* $\vec{B} = -2\hat{i} - 3\hat{j}$

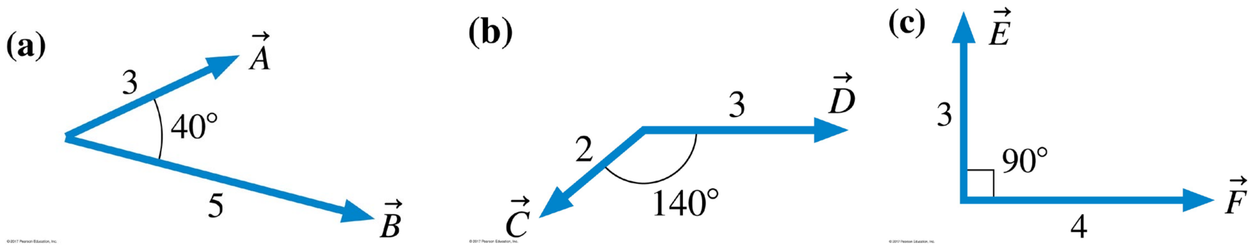
(b) $\vec{A} = 4\hat{i} + 2\hat{j}$ *and* $\vec{B} = 2\hat{i} + 3\hat{j}$

9.11. Solve: (a) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (4)(-2) + (-2)(-3) = -2.$

(b) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (-4)(2) + (2)(4) = 0.$

Problem 9.15

Evaluate the dot product of the three pairs of vectors in the figure below.



9.15. Solve: (a) $\vec{A}\cdot\vec{B} = AB\cos\alpha = (5)(3)\cos 40° = 11.$

(b) $\vec{C}\cdot\vec{D} = CD\cos\alpha = (2)(3)\cos 140° = -4.6.$

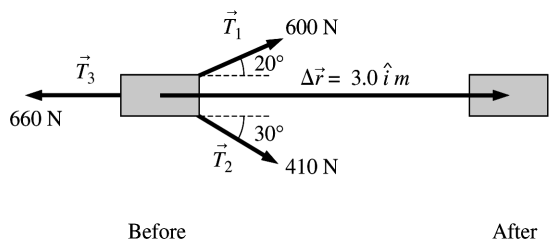
(c) $\vec{E}\cdot\vec{F} = EF\cos\alpha = (3)(4)\cos 90° = 0.$

Problem 9.19

The three ropes shown in the bird's-eye view of the figure below are used to drag a crate 3.0 m across the floor. How much work is done by each of the three forces?

9.19. Model: Model the crate as a particle and use $W = \vec{F}\cdot\Delta\vec{r}$, where W is the work done by a force $\vec{F}$ on a particle and $\Delta\vec{r}$ is the particle's displacement.

Visualize:



Solve: For the tension $\vec{T}_1$:

$$W = \vec{T}_1\cdot\Delta\vec{r} = (T_1)(\Delta r)\cos(20°) = (600\text{ N})(3.0\text{ m})(0.9397) = 1.7\text{ kJ}$$

For the tension $\vec{T}_2$:

$$W = \vec{T}_2\cdot\Delta\vec{r} = (T_2)(\Delta r)\cos(30°) = (410\text{ N})(3.0\text{ m})(0.866) = 1.1\text{ kJ}$$

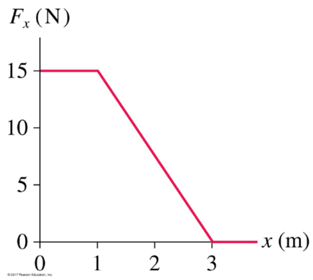
For the force $\vec{T}_3$:

$$W = \vec{T}_3\cdot\Delta\vec{r} = (T_3)(\Delta r)\cos(180°) = (660\text{ N})(3.0\text{ m})(-1.0) = -2.0\text{ kJ}$$

Assess: Negative work done by the force of kinetic friction $\vec{T}_3$ means that 1.95 kJ of energy has been transferred *out* of the crate.

Problem 9.21

A 500 g particle moving along the x-axis experiences the force shown in the figure below. The particle's velocity is 2.0 m/s at x = 0 m. What is its velocity at x = 3 m?



9.21. Model: Use the work–kinetic energy theorem to find velocities.
Solve: The work–kinetic energy theorem is

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = W = \int_{x_i}^{x_f} F_x dx = \text{ area under the force curve from } x_i \text{ to } x_f$$

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}(0.500\text{ kg})(2.0\text{ m/s})^2 = \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = \int_{x_i}^{x_f} F_x dx$$

$$\text{At } x = 1\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = 15\text{ N}\cdot\text{m} \Rightarrow v_f = 8.0\text{ m/s}$$

$$\text{At } x = 2\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = (15 + 11.25)\text{ N}\cdot\text{m} \Rightarrow v_f = 10\text{ m/s}$$

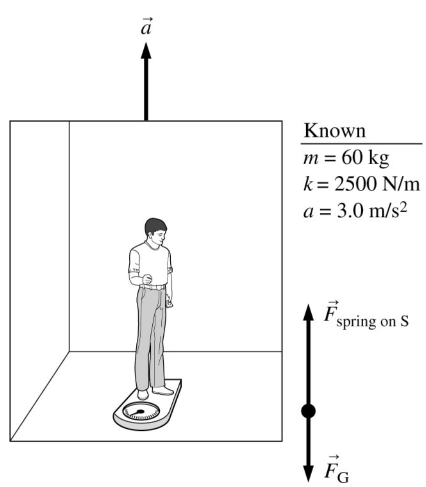
$$\text{At } x = 3\text{ m: } \tfrac{1}{2}mv_f^2 - 1.0\text{ J} = (15 + 11.25 + 3.75)\text{ N}\cdot\text{m} \Rightarrow v_f = 11\text{ m/s}$$

Assess: The speed is increasing as more work is done on it, but not linearly.

Problem 9.28

A 60 kg student is standing atop a spring in an elevator as it accelerates upwards at 3.0 m/s^2. The spring constant is 2500 N/m. By how much is the spring compressed?

9.28. Model: Model the student (S) as a particle and the spring as obeying Hooke's law.
Visualize:



Solve: According to Newton's second law the force on the student is

$$\Sigma(F_{\text{on S}})_y = F_{\text{spring on S}} - F_G = ma_y$$

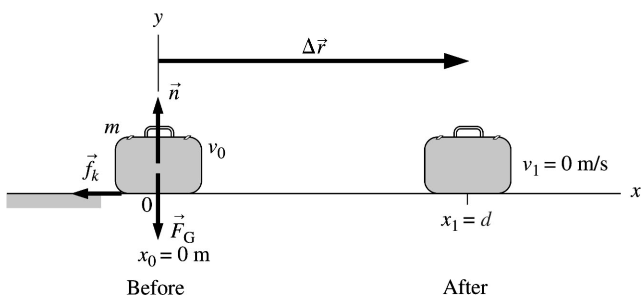
$$\Rightarrow F_{\text{spring on S}} = F_G + ma_y = mg + ma_y = (60\ \text{kg})(9.8\ \text{m/s}^2 + 3.0\ \text{m/s}^2) = 768\ \text{N}$$

Since $F_{\text{spring on S}} = F_{\text{S on spring}} = k\Delta y$, $k\Delta y = 768$ N. This means $\Delta y = (768\ \text{N})/(2500\ \text{N/m}) = 0.31$ m.

Problem 9.33

A baggage handler throws a 15 kg suitcase along the floor of an airplane luggage compartment with a speed of 1.2 m/s. The suitcase slides 2.0 m before stopping. Use work and energy to find the suitcase's coefficient of kinetic friction on the floor.

9.33. Model: Model the suitcase as a particle, use the model of kinetic friction, and use the work–kinetic energy theorem.
Visualize:



The net force on the suitcase is $\vec{F}_{\text{net}} = \vec{f}_k$.

Solve: The work–kinetic energy theorem gives

$$W_{\text{net}} = \Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 \quad\Rightarrow\quad \vec{F}_{\text{net}} \cdot \Delta\vec{r} = \vec{f}_k \cdot \Delta\vec{r} = 0\ \text{J} - \tfrac{1}{2}mv_0^2$$

$$(f_k)d\cos(180°) = -\tfrac{1}{2}mv_0^2$$

$$-\mu_k mgd = -\tfrac{1}{2}mv_0^2 \quad\Rightarrow\quad \mu_k = \frac{v_0^2}{2gd}$$

Inserting the given quantities into the expression for the coefficient of kinetic friction gives

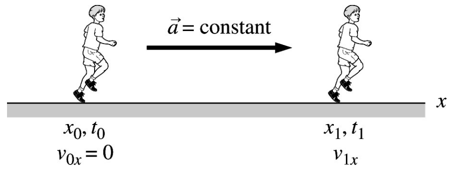
$$\mu_k = \frac{v_0^2}{2gd} = \frac{(1.2\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)(2.0\ \text{m})} = 0.037$$

Assess: Friction transforms kinetic energy of the suitcase into thermal energy. In response, the suitcase slows down and comes to rest. Notice that the coefficient of friction does not depend on the mass of the object, which is reasonable.

Problem 9.40

A 50 kg sprinter, starting from rest, runs 50 m in 7.0 s at constant acceleration. **(a)** What is the magnitude of the horizontal force acting on the sprinter? **(b)** What is the sprinter's power output at 2.0 s, 4.0 s, and 6.0 s?

9.40. Model: Model the sprinter as a particle, and use the constant-acceleration kinematic equations and the definition of power in terms of velocity.
Visualize:



Solve: **(a)** We can find the acceleration from the kinematic equations and the horizontal force from Newton's second law. We have

$$x = x_0 + v_{0x}(t_1 - t_0) + \tfrac{1}{2}a_x(t_1 - t_0)^2 \quad \Rightarrow \quad 50 \text{ m} = 0 \text{ m} + 0 \text{ m} + \tfrac{1}{2}a_x(7.0 \text{ s} - 0 \text{ s})^2 \quad \Rightarrow \quad a_x = 2.04 \text{ m/s}^2$$

$$F_x = ma_x = (50 \text{ kg})(2.04 \text{ m/s}^2) = 10 \times 10^1 \text{ N}$$

(b) We obtain the sprinter's power output by using $P = \vec{F} \cdot \vec{v}$, where $\vec{v}$ is the sprinter's velocity. At $t = 2.0$ s the power is

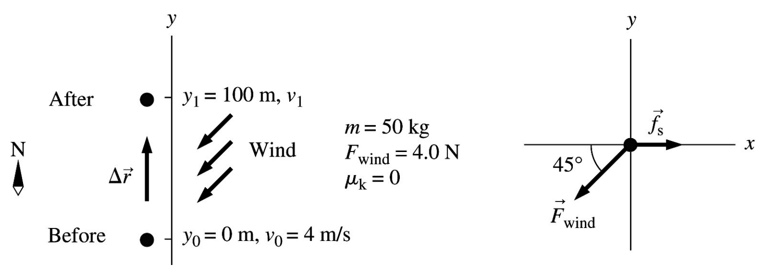
$$P = (F_x)[v_{0x} + a_x(t - t_0)] = (102 \text{ N})[0 \text{ m/s} + (2.04 \text{ m/s}^2)(2.0 \text{ s} - 0 \text{ s})] = 0.42 \text{ kW}$$

The power at $t = 4.0$ s is 0.83 kW, and at $t = 6.0$ s the power is 1.3 kW.

Problem 9.49

A 50-kg ice skater is gliding along the ice, heading due north at 4.0 m/s. The ice has a small coefficient of static friction, to prevent the skater from slipping sideways, but $\mu_k = 0$. Suddenly, a wind from the northeast exerts a force of 4.0 N on the skater. **(a)** Use work and energy to find the skater's speed after gliding 100 m in this wind. **(b)** What is the minimum value of μ_s that allows her to continue moving straight north?

9.49. Model: Use the particle model for the ice skater, the friction model, and the work–kinetic energy theorem.
Visualize:



Solve: **(a)** The work–kinetic energy theorem gives

$$\Delta K = \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 = W_{net} = W_{wind}$$

There is no kinetic friction along her direction of motion. Static friction acts to prevent her skates from slipping sideways on the ice, but this force is perpendicular to the motion and does not contribute to a change in thermal energy. The angle between $\vec{F}_{wind}$ and $\Delta\vec{r}$ is $\theta = 135°$, so

$$W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = F_{wind}\Delta y \cos(135°) = (4.0 \text{ N})(100 \text{ m})\cos(135°) = -282.8 \text{ J}$$

Thus, her final speed is

$$v_1 = \sqrt{v_0^2 + \frac{2W_{wind}}{m}} = 2.2 \text{ m/s}$$

(b) If the skates don't slip, she has no acceleration in the x-direction and so $(F_{net})_x = 0$ N. That is:

$$f_s - F_{wind}\cos(45°) = 0 \text{ N} \quad \Rightarrow \quad f_s = F_{wind}\cos(45°) = 2.83 \text{ N}$$

Now there is an upper limit to the static friction: $f_s \le (f_s)_{max} = \mu_s mg$. To not slip requires

$$\mu_s \ge \frac{f_s}{mg} = \frac{2.83 \text{ N}}{(50 \text{ kg})(9.8 \text{ m/s}^2)} = 0.0058$$

Thus, the minimum value of μ_s is 0.0058.

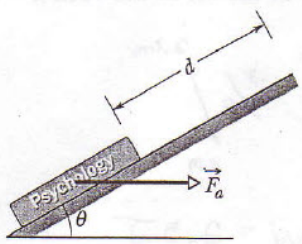
Problem 9.60
A 90 kg firefighter needs to climb the stairs of a 20-m-tall building while carrying a 40 kg backpack filled with gear. How much power does he need to reach the top in 55 s?

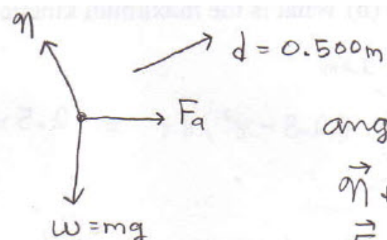
9.60. Model: The firefighter is a particle.
Visualize: The work done by the firefighter will equal the negative of the work the force of gravity does.
Solve: The power is the work done divided by the time it takes.

$$P = \frac{W}{\Delta t} = \frac{mg\Delta y}{\Delta t} = \frac{(90\text{ kg} + 40\text{ kg})(9.8\text{ m/s}^2)(20\text{ m})}{55\text{ s}} = 460\text{ W}$$

Assess: This power output is heavy exertion, but reasonable in the line of duty of a firefighter.

Problem 9.A
In the figure below, a horizontal force $\vec{F}_a$ of magnitude 20.0 N is applied to a 3.00 kg psychology book as the book slides a distance d = 0.500 m up a frictionless ramp at angle θ = 30.0°. (a) During the displacement, what is the net work done on the book by $\vec{F}_a$, the gravitational force on the book, and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is its speed at the end of the displacement?





angle between:
$\vec{n} + \vec{d} \rightarrow \phi = 90°$
$\vec{F}_a + \vec{d} \rightarrow \phi = 30.0°$
$\vec{F}_g + \vec{d} \rightarrow \phi = 120°$

(a) $W_{net} = W_n + W_{F_a} + W_g$

$= n\,d\cos 90° + F_a\,d\cos 30.0° + (mg)\,d\cos 120°$

$= 0 + (20.0\text{N})(0.500\text{m})\cos 30.0° + (3.00\text{kg})(9.80\text{m/s}^2)(0.500\text{m})\cos 120°$

$\boxed{W_{net} = 1.31\text{J}}$

(b) $W_{net} = K_f - \overset{\approx 0}{K_i} \rightarrow W_{net} = \frac{1}{2} m v_f^2 \rightarrow V_f = \sqrt{\frac{2W_{net}}{m}} = \sqrt{\frac{2(1.31\text{J})}{3.00\text{kg}}}$

$\boxed{V_f = 0.935\text{m/s}}$